Credit Default Swap Valuation: An Application to Spanish Firms

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Abstract

This paper presents and tests a model to price Credit Default Swaps (CDS) using the credit risk information extracted from the firms' bond market prices. Six Spanish firms are used in the analysis (BBVA, Caja Madrid, Endesa, Repsol YPF, SCH and Telefonica), which derives daily prices for the firms' CDS covering the period from April 2001 to April 2002. The model is shown to produce CDS prices more volatile than market quotes and to perform better the higher the firm's credit quality. Additionally, it is shown that the choice of the recovery rate does not affect the model implied CDS prices. The model is simple, the calibration and pricing procedure is quick, and the paper is intended as an introduction to credit derivatives pricing.

Keywords: credit default swap, Spanish firms, valuation.

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This paper is part of a series of surveys on credit risk modelling and pricing. The complete list of surveys is available at `www.abelelizalde.com`, and consists on the following:

1. Credit Risk Models I: Default Correlation in Intensity Models.
2. Credit Risk Models II: Structural Models.
3. Credit Risk Models III: Reconciliation Reduced-Structural Models.
4. Credit Risk Models IV: Understanding and pricing CDOs.
5. Credit Default Swap Valuation: An Application to Spanish Firms.

1 Introduction

Credit risk has been attracting a great deal of attention recently. Risk-management practitioners and regulators have all been working intensively on the development of methodologies and systems to quantify credit risks. At the same time, the market for credit derivatives has grown rapidly in recent years, and it is expected to continue growing. Credit Default Swaps (CDS) have become the most liquid credit derivative instruments. In terms of outstanding notional, they represent around 85% of the credit derivatives market, which has a total outstanding notional in excess of USD 4000 billions. Not only are they the most important credit derivative instrument, but they are also the building block for many of the other more exotic structures traded in the credit derivatives market.

Briefly, CDS are instruments by which an investor, referred to as protection buyer, buys protection against the losses derived from the default of a given firm. The party which acts as insurer in a CDS contract, referred to as protection seller, receives, from the protection buyer, a periodic fee as payment for the insurance. In return for such payments, the protection seller is responsible to, in case of default of the firm, reimburse the protection buyer the losses derived.

In this paper we apply a risk neutral model to price CDS of six different Spanish firms (BBVA, Caja Madrid, Endesa, Repsol YPF, SCH and Telefonica) from April 2001 to April 2002, and compare the prices derived from the model with market prices, in order to test its pricing performance.

Two main types of models have been used in the credit risk literature: structural and reduced (or intensity) models. Structural models derive default probabilities from the evolution of the firms' structural variables such as the firms' assets and liabilities structure. Reduced models rely on the firms' defaultable securities market prices to extract the evolution of the firms' credit risk, focusing on the default probabilities priced by those markets.

We rely on reduced form models to derive the pricing formulas used throughout the paper, mainly because of their easy calibration from market data. After deriving the pricing formulas both for bonds and CDS, we use a two step procedure to price CDS. First, we use each firm's bond prices to estimate the firm's default probabilities and, second, we use them to obtain CDS prices.

The recovery rate is defined as the value of the firm's bond prices just after default, expressed as a percentage of the bonds face value. It is assumed to be a constant and exogenously given parameter. The recovery rate is a key ingredient in the CDS pricing formula, because it determines the payment of the protection seller to the protection buyer in case of default. We show that the choice of the recovery rate does not significantly affect the model implied CDS prices, as long at it lies in a reasonable interval (between 20% and 60%).

Additionally, we show that model implied CDS prices are more volatile than market prices and that the model works better for firms with smaller credit risk.

The remainder of this paper is structured as follows. The rest of the introductory Section contains a review and introduction of credit derivatives (CDS in particular), the current state and evolution of credit derivatives markets, a brief comparison of different credit risk models used in the literature, and, finally, a review of other research papers dealing with the analysis and pricing of CDS. Section 2 presents the model and derives the pricing formulas used in the process of pricing CDS. Section 3 describes the dataset, and Section 4 presents the results. The paper ends with some concluding remarks.

1.1 Credit derivatives

Credit derivatives allow investors to trade the credit risk inherent in a given defaultable instrument without having to trade the instrument itself. As a consequence, market participants can modify their credit risk exposure to any given firm or country (or a set of them) without having to buy or sell defaultable instruments, usually debt securities, issued by such a company or country.[1]

In a typical credit derivatives contract, one party, the *protection buyer*, buys protection against the credit risk of a given issuer (or group of issuers[2]), referred to as *reference entity*. The other party, the *protection seller*, will bear the losses derived from the default of the reference entity. In the trade between the protection seller and the protection buyer there is a priory no exchange of any defaultable instrument issued by the reference entity: they only trade the credit risk of that entity. If the reference entity defaults, the protection seller will reimburse the protection buyer the losses, caused by the default, in some pre-determined bond of the reference entity. That pre-determined bond is called *reference obligation*.

Any credit derivatives contract has to specify the reference entity (or entities), the maturity of the contract, the exact definition of default (which will trigger the payment from the protection seller to the protection buyer), the way the losses derived from the reference entity's default are measured, ...

Most credit derivatives use a set of standard definitions for the above characteristics. The publication of the *Credit Derivatives Definitions* by the International Swap and Derivatives Association (ISDA) in 1999 was a big step towards standardizing the terminology in credit derivatives transactions. The ISDA Definitions established a uniform set of definitions of important terms, such as the range of credit events that could trigger payments or deliveries. The definitions increased flexibility and reduced the complexity of administration and documentation.

For example, the default of the reference entity, also called *credit event*, is most commonly defined as the occurrence of one or more of the following: failure to meet payment obligations when due, bankruptcy or moratorium, repudiation, material

[1] See J.P. Morgan (1999) for a more detailed description. Saunders and Allen (2002) contains an introductory chapter on credit derivatives.

[2] In this paper we focus on credit derivatives which deal only with the credit risk of a single issuer. These are called *single-name* credit derivatives, as opposed to *multi-name* credit derivatives, which deal with the credit risk of a given group of issuers. We refer the reader interested in multi-name credit derivatives to, for example, Gregory (2003), although there are plenty of books in the market dealing with this issue.

adverse restructuring of debt, obligation acceleration or obligation default.[3]

1.1.1 Credit Default Swaps (CDS)

A CDS is a credit derivative in which there is only one reference entity, i.e. a single-name credit derivative. The face value or notional of the CDS is the face value of the reference obligation whose credit risk is being insured.

The buyer of protection in a CDS is insured against the default of the reference entity. The protection seller receives a periodic fee, referred to as CDS premium, for taking the credit risk. The protection buyer pays the premium until either the expiration of the contract or the default of the reference entity, whichever first. If default occurs, the buyer is typically required to pay the part of the premium payment that has accrued since the last payment date; this is called the *accrual payment*.

In the event of default, the protection seller must compensate the buyer for the loss in the value of the reference obligation caused by the default. This compensation can be implemented through two *settlement mechanisms*: cash or physical settlement. Cash settlement consists of a cash payment (from the protection seller to the protection buyer) equal to the fall in price of the reference obligation below par after the default.[4] In a physical settlement, the protection buyer delivers the protection seller a portfolio of the reference entity's obligations (with face value equal to the CDS notional) and receives in cash, from the protection seller, their face value. The obligations that can be delivered by the protection buyer are called *deliverable obligations* and may be the reference obligation or one of a broad class of obligations meeting certain specifications, usually in terms of seniority and maturity. Since, after a credit event, the price of the deliverable obligations will probably differ, the protection buyer will deliver the cheapest of all deliverable obligations. This latter feature is commonly referred to as the *delivery option*. The lower the range of deliverable

[3] Packer and Zhu (2005) show how different definitions of credit event affect CDS prices.

[4] When there is cash settlement, the calculation agent polls dealers to determine the mid-market price of the reference obligation some specified number of days after the credit event. Alternatively, counterparties can fix the contingent payment as a predetermined sum, known as a binary settlement.

obligations, the lower the value of the delivery option for the protection buyer.

When the parties enter into a CDS contract there is no initial payment involved: the CDS premium is fixed in such a way that the market value of the CDS at origination is zero.

1.1.2 Credit derivatives market

The credit derivatives market is relatively small compared to other, more mature, derivatives markets such as interest rate and currency derivatives markets.[5] However, it is growing rapidly, reflecting the fact that credit derivatives have proven to be a very useful way of managing the relatively large and growing volumes of credit risk that global markets deal with on a daily basis.

Recent data published by the Bank for International Settlements (BIS)[6] show that credit derivatives are the fastest growing segment of the global OTC derivatives market, growing more than sixfold from 2001 to 2004. The notional amount outstanding in the credit derivatives markets was, in USD billions, 118 in June 1998, 695 in June 2001, and 4477 in June 2004.[7] ISDA estimates that the figure for 2004 is more than 8000 USD billions. Reliable numbers are hard to come by because trading occurs over the counter. According to BIS: "The increase was concentrated in credit default swaps (CDS), the use of which was driven by the standardization of contractual terms, the emergence of CDS indices and trading platforms and the issuance of collateralized debt obligations (CDO)."

While CDS represented around 50% of the credit derivatives market in 2000 according to British Bankers' Association (2000), their importance in the credit derivatives market has grown to around 85% in 2004. CDS have become so popular that the

[5] The credit derivatives market represented, at the end of 1999, around one percent of the underlying notional value of the global volume of OTC derivatives, according to British Bankers' Association (2000).

[6] See BIS (2002 and 2004). The BIS survey covers 52 countries, including the top 10 industrialized countries. Dealers report their transactions to the BIS via their own central bank.

[7] Notional amounts outstanding are defined as the nominal value of all deals concluded and not yet settled at the reporting date.

Financial Times reported in May 6, 2005, the launching by AXA of the first managed CDS fund aimed at retail investors.

With respect to maturities, the most liquid CDS contract has a five years maturity, followed by the three year and then the one and ten year maturities.

1.1.3 Credit risk models

There are two primary types of models that attempt to describe default processes in the credit risk literature: structural and reduced (or intensity) form models.[8]

Structural models use the evolution of firms' structural variables, such as asset and debt values, to model the time of default. Merton's model (1974) was the first modern model of default and is considered the first structural model. In Merton's model, a firm defaults if, at the time of servicing the debt, its assets are below its outstanding debt. A second approach, within the structural framework, was introduced by Black and Cox (1976). In this approach defaults occur as soon as firm's asset value falls below a certain threshold. In contrast to the Merton approach, default can occur at any time.

Structural default models provide a link between the credit quality of a firm and the firm's economic and financial conditions and, as a consequence, defaults are endogenously generated within the mode.

Reduced form models do not consider the relation between default and firm's assets and liabilities structure in an explicit manner. Default probabilities are obtained from the credit risk inherent in the prices of the firms' traded defaultable securities. The reduced form approach was introduced by, among others, Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Madam and Unal (1998), Duffie and Singleton (1999) and Kijima and Muromachi (2000).

The empirical testing of structural models in general has not been very successful: they tend to underpredict credit risk.[9] In contrast, the genius of reduced form models

[8] Elizalde (2005a) and (2005b) present, respectively, a review and survey of the reduced and structural credit risk models literature.

[9] See Anderson and Sundaresan (2000), Eom, Helwege, Huang (2003), and Ericsson and Reneby

is their easy implementation and calibration.

1.2 Related literature

Although the literature in credit risk modeling is well established, there is a limited number of studies looking at CDS pricing. Duffie (1999) represents one of the first attempts to derive the basics of CDS pricing, although the model is not tested against market data.

Hull and White (2000) provide a similar methodology for pricing CDS but, again like in Duffie (1999), the model is not directly tested against CDS market data. The authors analyze the effects of the assumed recovery rate on the CDS prices and obtain that, if the same recovery rate is used both for estimating default probabilities and for pricing CDS using those probabilities, the chosen recovery rate has little impact on the implied CDS premium as long as the recovery rate is assumed to be lower than 50% of the bond's face value.

Hull and White (2001) extend Hull and White (2000) to consider the possibility of the CDS protection seller defaulting during the life of the CDS.[10] They show that the impact of the protection seller default risk on CDS premiums depends both on the credit quality of the protection seller and on the correlation between the default probabilities of the reference entity and the protection seller. When that correlation is zero, the impact protection seller default risk is very small. However, it has a bigger effect as the correlation increases and the credit quality of the protection seller declines. Again, the paper does not empirically test whether the performance of the model to price CDS improves when protection seller default risk is considered.

The idea for our research comes from Houweling and Vorst (2004), which represents, to the best of our knowledge, the first paper to empirically test the ability of a reduced credit risk model to price CDS. We apply their model and pricing procedure to the Spanish case using a more recent data period. Houweling and Vorst (2004)

(2004).

[10] Chen and Sopranzetti (2002) represent another attempt to analyze the impact of the protection seller default probability into, among other credit derivative contracts, CDS.

show that the model generates more accurate predictions about CDS premiums than using bonds' credit spreads over the default-free rate as an approximation to CDS premiums. The authors also show that, as it is the case in this paper, the model generates CDS premiums relatively insensitive to the assumed recovery rate.

Some papers have examined the dynamics and determinants of CDS prices. Skinner and Townend (2002) argue, by appealing to the put-call parity, that a CDS can be expressed as a put option on the reference obligation. They find that variables affecting option prices such as default-free rates, volatility, underlying asset and time to maturity are also important in determining CDS prices.

Aunon-Neri et al. (2002) find that fixed-income related variables (ratings, interest rates, bond spreads, ...) and equity related variables (stock prices, asset volatility, leverage, ...) explain up to 82% of the variation in CDS premiums, being ratings the most important single source of information on credit risk overall.

Blanco, Brennan and Marsh (2004) show that the CDS market leads the bond market in determining the price of credit risk, and that CDS prices are a cleaner indicator of credit risk than bond spreads. They also find that, on average, the bond and CDS markets price credit risk equally. This fact supports the pricing strategy followed in this paper since we use the credit risk priced in the bond market to estimate the firms' default probabilities which are then used in the CDS pricing.

Hull, Pedrescu and White (2004) find evidence that negative announcements by rating agencies (downgrades, review for downgrade and negative outlook) are anticipated by CDS prices.

2 Model

We follow a two-step procedure to price CDS. First, we derive, for every day, and using each firm's bond market prices, the firm's default probabilities. Second, we use such probabilities to price CDS of each firm.

To implement the model we need pricing formulas for both the bond and CDS prices. We use a very simple reduced form model, which will make the default probability estimation and CDS pricing extremely quick, at the cost of the following set of assumptions:

Assumption 1. The spread between default-free and corporate bonds is due, exclusively, to credit risk.

Though bonds spread over default-free short rates may account for other risks apart from credit risk, for example liquidity risk and tax considerations, we will ignore them.[11]

Assumption 2. Constant and exogenous recovery rate.

The recovery rate is the value of the bonds immediately after default, expressed as a percentage of the bonds' face value.[12] Ideally, one would like to estimate the recovery rate priced in the bonds' data together with the firm's default probabilities. However, default probabilities and recovery rate can not be jointly identified from bond data. As a consequence, the recovery rate is fixed to identify the model.

Table 1 presents estimates obtained from Moody's Investor's Service (2000). Recovery rates lie between 15% and 60% of the bond's face value and are an increasing function of the seniority of the bond.[13]

Bond seniority	Mean (%)	Std. Dev.
Senior secured	56.3	25.1
Senior unsecured	48.8	25.0
Senior subordinated	39.4	24.5
Subordinated	33.1	20.7
Junior Subordinated	16.6	13.8

Table 1. Recovery Rates on Corporate Bonds. Percentage of the bonds' face value. Source: Moody's Investor's Service (2000).

[11] Amato and Remolona (2003) provide a detailed analysis of the different components behind corporate bonds spreads and the literature which studies the relative weight of such components (default risk, loss given default, risk premium, liquidity spread and tax component).

[12] This definition of the recovery rate receives the name of recovery of face value. See Lando (2004, Chapter 5) and Elizalde (2005a) for alternative ways of modelling the recovery rate in reduced form models.

[13] Altman and Arman (2002) find similar values.

Assumption 3. Complete market and absence of arbitrage opportunities.

For our purposes we shall use the class of equivalent probability measures $\mathbf{P}$, where non-dividend paying asset processes discounted using the default-free short rate r_t are martingales. Absence of arbitrage is a necessary requirement for the existence of at least one equivalent probability measure, and the assumption of market completeness guarantees its uniqueness. Such an equivalent measure is called a *risk neutral measure* and will be used to derive bonds and CDS pricing formulas.[14]

Assumption 4. Independence of the firm's credit quality and the default-free interest rates under the risk neutral probability measure.

Although the model could accommodate correlation between interest rates and survival probabilities, such an assumption would add a high degree of complexity into the model because a process would have to be estimated for the default-free short rate as well.[15]

Assumption 5. Neither the protection seller nor the protection buyer default during the life of the CDS.

While the protection seller default risk would reduce the value of the CDS for the protection buyer, reducing the premium, the protection buyer default risk would have the opposite effect. The net effect would depend on the default risk level of both agents and the credit risk correlations between them and the reference entity.

2.1 Pricing formulas

This Section builds on Houweling and Vorst (2004). In reduced form models, defaults are assumed to occur unexpectedly: the time of default is modelled as the first arrival of a Poisson process with a given intensity or hazard rate λ_t, which represents the instantaneous default probability of a firm which has not defaulted before t. If we

[14] See Elizalde (2005a) for an analysis of the different scenarios under which the transition from the physical to the equivalent (or risk neutral) probability measure can be accomplished.

[15] Elizalde (2005c) estimates a reduced form model in which default probabilities depend on default-free interest rates. The results show that the effect of default-free rates on default probabilities is small. Moreover, different empirical papers find different signs for that effect.

denote by τ the time of default of a given firm, the hazard rate at time $t \leq \tau$ is given by

$$\lambda_t = \lim_{h \to 0} \frac{\mathbf{P}\left[\tau \in (t, t+h] \mid \tau > t\right]}{h}, \tag{1}$$

and the probability at time t that the firm will survive until some future time T is given by $s(t,T)$:[16]

$$s(t,T) = \mathbf{P}\left[\tau > T \mid \tau > t\right] = E\left[\exp\left(-\int_t^T \lambda_s ds\right)\right]. \tag{2}$$

Reduced form models rely on the dynamics of the hazard rate in order to capture the evolution of the firms' default risk.[17]

2.1.1 Bond pricing formula

The value $\beta(t,T)$ at time t of a default-free zero-coupon bond with maturity T and face value 1 can be expressed as

$$\beta(t,T) = E\left[\exp\left(-\int_t^T r_s ds\right)\right]. \tag{3}$$

Consider a defaultable zero coupon bond with maturity T and face value 1 that, in case of default at time $\tau < T$, generates a recovery payment of a fraction δ of the bond's face value, where δ is the recovery rate, assumed constant and exogenously given. The price of such a bond at time $t < T$ is given by

$$z(t,T) = E\left[\beta(t,T)\mathbf{1}_{\{\tau > T\}}\right] + E\left[\beta(t,\tau)\delta\right], \tag{4}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function.

Using the independence of default-free interest rates and the default time we obtain the following expression for the price, at time t, of a defaultable zero-coupon bond with maturity T and face value 1:

[16]The default probability would be given by $1 - s(t,T)$.

[17]During all the paper we make use of the risk neutral probability measure $\mathbf{P}$ and, therefore, all hazard rates and implied survival probabilities are given under that measure.

$$
\begin{aligned}
z(t,T) &= \beta(t,T)\,E\left[\mathbf{1}_{\{\tau>T\}}\right] + E\left[\beta(t,\tau)\right]\delta && (5)\\
&= \beta(t,T)\,s(t,T) + \delta\int_t^T \beta(t,s)\,f(s)\,ds, && (6)
\end{aligned}
$$

where $f(t)$ denotes the probability density function associated with the intensity process λ_t :

$$
f(t) = \lambda_t \exp\left(-\int_0^t \lambda_s ds\right). \tag{7}
$$

Finally, consider a defaultable coupon bond with payment dates $\{t_1, ..., t_N\}$, coupon c (expressed as a percentage of the bond's face value), and face value M. Its price $z(t_0, t_N, c)$ at time $t_0 < t_1$ equals the sum of the expected discounted value of its coupons and face value and a potential recovery payment in case of default. The i^{th} coupon payment is only made if the bond issuer has not yet gone bankrupt at time t_i. Similarly, the face value is only paid if the bond is still alive at time t_N. If the bond defaults at time τ before maturity, a recovery payment of δM is made. Therefore

$$
\begin{aligned}
z(t_0,t_N,c) &= \sum_{i=1}^{N}\beta(t_0,t_i)\,E\left[\mathbf{1}_{\{\tau>t_i\}}\right]cM + \beta(t_0,t_N)\,E\left[\mathbf{1}_{\{\tau>t_N\}}\right]M && (8)\\
&\quad + E\left[\beta(t_0,\tau)\,\mathbf{1}_{\{\tau\le t_N\}}\right]\delta M, && (9)
\end{aligned}
$$

which evaluating the expectations becomes

$$
z(t_0,t_N,c) = \sum_{i=1}^{N}\beta(t_0,t_i)\,s(t_0,t_i)\,cM + \beta(t_0,t_N)\,s(t_0,t_N)\,M + \delta M\int_{t_0}^{t_N}\beta(t_0,s)\,f(s)\,ds, \tag{10}
$$

Following Houweling and Vorst (2004), in the empirical application, the integral expression

$$
\int_{t_0}^{t_N}\beta(t,s)\,f(s)\,ds, \tag{11}
$$

is replaced by a numerical approximation. Let $(h_0, ..., h_m)$ be a monthly grid of maturities such that $h_0 = t_0$ and $h_m = t_N$. The numerical approximation is given by

$$
\int_{t_0}^{t_N}\beta(t_0,s)\,f(s)\,du \approx \sum_{j=1}^{m}\beta(h_0,h_j)\left(s(t,h_{j-1}) - s(t,h_j)\right). \tag{12}
$$

2.1.2 CDS pricing formula

Similar to a plain vanilla interest rate swap, a CDS consists of two legs: a fixed and a floating leg. The fixed leg represents the payments from the protection buyer to the protection seller, whereas the floating leg represents the payments from the protection seller to the protection buyer.

Consider a CDS with payment dates $\{t_1, ..., t_N\}$, maturity t_N, premium percentage p and notional M. The contract starts at time $t_0 < t_1$, and the first premium is due at t_1.

Value of the fixed leg X_F

At each payment date t_i, for $i = 1, ..., N$, the protection buyer pays to the protection seller an amount

$$pM\alpha\left(t_{i-1}, t_i\right), \tag{13}$$

where $\alpha\left(t_{i-1}, t_i\right)$ is the year fraction between t_{i-1} and t_i. If the reference entity does not default during the life of the contract, the buyer makes all scheduled payments. However, if default occurs at time $\tau \leq t_N$, the buyer has made only $I\left(\tau\right)$ payments, where

$$I\left(\tau\right) = \max\left(i = 0, ..., N : t_i < \tau\right) \tag{14}$$

and the remaining payments $I\left(\tau + 1\right), ..., N$ are no longer due. In addition, he has to make an accrual payment of $\alpha\left(t_{I(\tau)}, \tau\right) p$ at time τ.[18] This accrual payment represents the payment due to the period between the last payment date $t_{I(\tau)}$ and the time of default.

[18] It is assumed that if default time exactly coincides with a payment date t_i, the buyer does not make regular payments, but makes an accrual payment, *i.e.* $I\left(t_i\right) = i - 1$. Since the regular payment and the accrual payment are equal on payment date, this assumption does not affect the value of the CDS.

The value of the fixed leg at time t is thus equal to

$$\begin{aligned} X_F &= \sum_{i=1}^{N} \beta(t_0, t_i)\, \alpha(t_{i-1}, t_i)\, E\left[\mathbf{1}_{\{\tau > t_i\}}\right] pM + E\left[\beta(t_0, \tau)\, \alpha\left(t_{I(\kappa)}, \tau\right) \mathbf{1}_{\{\tau \le t_N\}}\right] pM \\ &= pM \left[\sum_{i=1}^{N} \beta(t_0, t_i)\, \alpha(t_{i-1}, t_i)\, s(t, t_i) + \int_{t_0}^{T_N} \beta(t_0, s)\, \alpha\left(t_{I(s)}, \tau\right) f(s)\, ds\right]. \quad (15) \end{aligned}$$

Value of the floating leg X_V

To compute the value of the floating leg, we must distinguish between cash settlement and physical settlement. If default occurs and the contract specifies cash settlement, the protection seller pays the buyer an amount equal to the difference between the reference obligation face value M and the final price of the reference obligation.[19] Under our recovery assumption, the final price of the reference obligation is a fraction δ of its face value M, so that the value of the floating leg under cash settlement is

$$X_V = E\left[\beta(t_0, \tau)(1 - \delta) M \mathbf{1}_{\{\tau \le t_N\}}\right] = (1 - \delta) M \int_{t_0}^{t_N} \beta(t_0, s) f(s)\, ds. \quad (16)$$

If the CDS contract specifies physical settlement at the default time, the buyer would deliver one or more deliverable obligations with a total notional of M to the seller, and the seller pays M to the buyer.[20] Assuming the only deliverable obligation is the reference obligation the net value of these transfers is equal to $(1 - \delta) M$, in which case the value of the floating leg under physical settlement is equal to its value under cash settlement. However, a CDS contract generally allows the buyer to choose from a list of deliverable obligations. Although our CDS dataset specifies physical settlement we use the value of the floating leg under cash settlement.

[19] The final price is the market value of the reference obligation on the default date as computed by an specified calculation agent by an specified valuation method. Commonly, one or more dealers are asked for quotes on the reference obligation; the highest and lowest quotes are disregarded and the arithmetic mean of the remaining quotes represents the value of the reference obligation.

[20] The market value of the reference obligation is δM, which would be the cost for the protection buyer of buying it in the market.

CDS premium

The premium p is chosen in such a way that the value of the CDS is equal to zero, because only then the value of the CDS to the protection buyer equals the value of the CDS to the protection seller and there is no need of any up-front payment:

$$X_F = X_V, \tag{17}$$

which implies

$$p = \frac{(1-\delta)\int_{t_0}^{t_N} \beta(t_0,s)\, f(s)\, ds}{\sum_{i=1}^{N} \beta(t_0,t_i)\,\alpha(t_{i-1},t_i)\, s(t,t_i) + \int_{t_0}^{T_N} \beta(t_0,s)\,\alpha\left(t_{I(s)},\tau\right) f(s)\, ds} \tag{18}$$

To numerically evaluate the integrals in the expressions appearing in (18), we use the same approximation as for the integral (11) in the defaultable coupon bond price.

2.2 Choice of hazard rate

Three different hazard processes are contemplated; all of them deterministic, one constant and two time-dependent.

	Hazard rate λ_t
Constant	a
Linear	$a + bt$
Quadratic	$a + bt + ct^2$

Table 2. Hazard rate specifications considered.

Other specifications could have been considered, such as, for example, the hazard rate implied by a Weibull distribution function. Although all of the previous distribution functions assume deterministic hazard rates, it might be more reasonable to assume stochastic hazard rates.[21] Apart from a higher complexity in the pricing formulas, a stochastic hazard rate requires a higher amount of bond data and makes the

[21] See Elizalde (2005a) for a detailed analysis of different stochastic specifications for the default hazard rate.

parameters estimation process more complex and time consuming. In this paper, we opt for the simplicity of the valuation formulas implied by the deterministic hazard rates contained in Table 2 as well as for the high computation speed when estimating their parameters.

3 Data

The study is going to be implemented daily using Spanish firms with traded CDS and enough number of bonds in order to estimate the hazard rate parameters. The time period goes from April 2001 to April 2002, and the firms analyzed are BBVA, Caja Madrid, Endesa, Repsol YPF, SCH and Telefonica. Table 3 describes the sector and rating of the firms considered.

Firm	Sector	Rating (S&P)
BBVA	Banks	AA
Caja Madrid	Banks	AA
Endesa	Utility	A
Repsol YPF	Basic Materials	A
SCH	Banks	A
Telefonica	Utility	A

Table 3. Firms' characteristics.

To estimate the credit risk models we construct a sample of fixed-coupon, senior bonds denominated in euros, with an outstanding amount higher than 300 million, annual coupon payments, between 1 and 15 years of maturity, and without convertibility or callability characteristics. The bond data set has been obtained from Bloomberg, and bond prices represent mid quotes. We only consider sufficiently liquid bonds, labelled as BGN by Bloomberg. As indicated by Blanco, Brennan and Marsh (2004), these are a weighted average of firm and indicative quotes submitted by at least five brokers or dealers. The exact weighting method is proprietary but firm quotes receive a higher weight than merely indicative quotes. In the period of study we had the following number of bonds for each firm: BBVA (5), Caja Madrid (4), Endesa (6), Repsol YPF (6), Telefonica (4) and SCH (4).

For BBVA, Endesa, SCH and Telefonica we have a large enough number of bond quotes for all the period considered, however for Repsol YPF we only have bond quotes since November 23, 2001 and for Caja Madrid since October 5, 2001.

Regarding default-free rates, the question of which of the available proxies (Treasury, repo or swap rates) is more appropriate remains open because of the differences in credit and liquidity components across proxies. Houweling and Vorst (2004) find that swap and repo rates overperform Treasury rates when used for fitting bond prices and pricing CDS, concluding that "the government curve is no longer seen as the reference default-free curve."[22] Following the results of Houweling and Vorst (2004) our proxy for default-free interest rates will be estimates of the zero-coupon euro and dollar curves using swap data. This dataset, provided by Banco de España (Bank of Spain), contains parameter estimates which allow us to construct the zero-coupon default-free rates for any maturity, which in turn are transformed into the default-free discount factors, or default-free zero-coupon bonds, given by (3). Data are constructed in the following way: for each day, the term structure is fitted using Svensson's procedure, which is a modification of the well-known parametrization of Nelson and Siegel (1987).[23] The result is a six-parameter set for each trading day, from which default-free zero-coupon rates (and bonds) of any maturity can be obtained.

Bond data are euro-denominated, so in the estimation of the hazard rate parameters the euro-denominated default-free curve is used. However, CDS are denominated in USD, thus to price them we use the dollar-denominated default-free curve.

The CDS data set contains mid prices from daily sheets posted by JP Morgan. All prices are based on a transaction with physical settlement and a notional amount

[22] Hull, Pedrescu and White (2004) contain a detailed analysis about alternative default-free rates, and their results are consistent with Houweling and Vorst (2004). See also Duffie and Singleton (1997) and Liu, Longstaff and Mandell (2002)

[23] See Nuñez (1995) and references therein.

of ten million USD. Each quote is given by a quarterly premium p expressed in basis points on an annual basis. The data set contains CDS quotes for each reference entity, with maturities of 1, 2, ..., 10 years for each day. The reference obligation is the reference entities' senior debt.

We need dates in which we have a large enough number of bond prices (at least three bond prices), CDS quotes and default-free interest rates estimates available. Restricting the data set with these constraints leaves 10680 CDS quotes. Table 4 shows this final data set.

Firm	Start date	End date	Number of Days
BBVA	25 April 01	19 April 02	208
Caja Madrid	5 October 01	19 April 02	116
Endesa	25 April 01	19 April 02	218
Repsol YPF	23 November01	19 April 02	89
SCH	25 April 01	19 April 02	217
Telefonica	25 April 01	19 April 02	220

Table 4. Final dataset.

Table 5 shows average CDS premiums by firm and maturity. Average CDS premiums grow monotonically with the maturity of the CDS for all firms. The firms' CDS premiums ranking can be fairly discerned. Repsol YPF is the firm with the highest CDS premiums for all the maturities. Telefonica and SCH comprise the second group of firms with higher average CDS premiums. The firms with the smallest average CDS premiums are Endesa, BBVA and Caja Madrid.

Maturity	Repsol YPF	Telefonica	SCH	Endesa	BBVA	Caja Madrid
1 year	138.7	37.4	18.6	15.2	7.2	10.3
2 years	157.0	56.7	34.5	20.5	11.3	13.8
3 years	163.9	63.6	40.1	22.8	13.0	15.4
4 years	167.9	73.2	54.5	29.7	20.8	18.5
5 years	171.2	79.5	63.8	34.6	25.8	20.8
6 years	173.5	87.2	72.4	39.5	28.8	23.8
7 years	175.9	93.4	78.8	43.8	31.6	26.6
8 years	177.8	99.4	85.9	49.1	33.8	28.6
9 years	179.8	104.2	91.5	53.4	35.5	30.5
10 years	180.9	108.3	95.4	56.9	37.0	32.2

Table 5. Average CDS premiums (in basis points) by firm and maturity.

Figure (1) represents the five year CDS premiums by firm during the sample period.[24] Repsol YPF has the most spectacular CDS premiums evolution: its quotes remain in the range from 50 to 150 basis points until December 2001, then they grow dramatically reaching levels of 400 basis points in February 2002 after which they decrease, although remaining much higher that those of the rest of the firms. In the same way Telefonica and SCH experience an increase in their CDS premiums during the last quarter of 2001. This pattern is also followed by BBVA, although in a smaller magnitude. Endesa, and Caja Madrid show a constant pattern of CDS premiums during the period studied.

4 Results

Since we want to evaluate whether the CDS pricing is sensitive to the choice of the recovery rate, all the valuation process will be made using nine different recovery rates: 10, 20, 30,..., 80 and 90%. For each day and firm, we have bond market prices with different coupons and maturities. Estimates of the hazard rate parameters are obtained minimizing (for each firm, day, recovery rate and hazard rate specification) the sum of the squared bond pricing errors between model and market prices. Those estimates are then used to compute CDS premiums. As a consequence, we obtain CDS premiums for each day using only the information contained in bond prices for that particular day.

4.1 Estimating hazard rates from bond prices

We estimate the models for each issuer and for each day we have at our disposal CDS quotes, default-free curve estimations and at least four bond quotes, with two exceptions: we use only three bonds for Caja Madrid from October 5, 2001 to February 21,

[24]The scale which measures the CDS premiums in all subplots is the same, from 0 to 140 basis points, except for Repsol YPF, in which it ranges from 0 to 450 basis points.

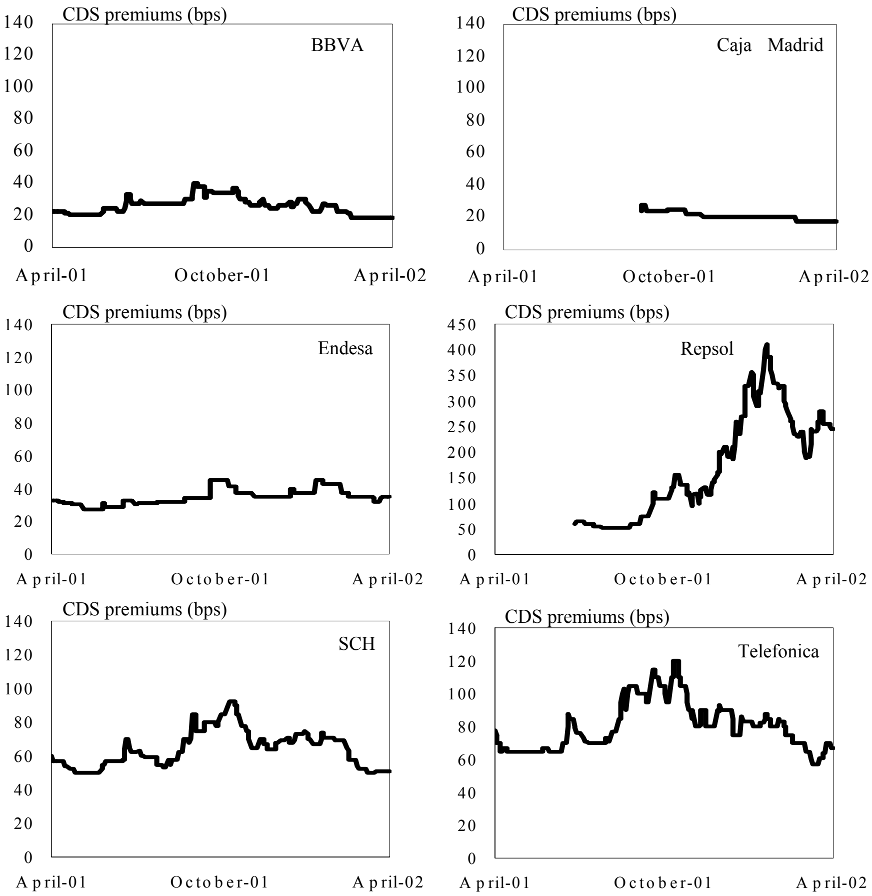

Figure 1: Five year CDS premiums, in basis points, by firm.

2002 and for Telefonica from April 25, 2001 to October 22, 2001. In these two cases the estimation is carried out only for constant and linear hazard rates, which imply one and two parameters respectively, but not for quadratic hazard rate.

The ability of each specification (hazard rate and recovery rate) to fit bond prices is assessed by looking at the root mean squared error (RMSE) of the deviations between market and model prices.

Table 6 contains RMSE by recovery and hazard rate. Linear and quadratic hazard rates clearly overperform the constant one, reflecting that the more parameters in the hazard function the better the fit.[25] Table 6 shows evidence that all recovery rates

[25] Since the quadratic hazard function implies one parameter more than linear hazard function, its

from 10% to 70% perform approximately equally in fitting the data.

	Constant	Linear	Quadratic
10%	2.61	0.67	0.71
20%	2.45	0.67	0.71
30%	2.39	0.67	0.71
40%	2.32	0.67	0.73
50%	2.10	0.68	0.75
60%	1.89	0.68	0.80
70%	1.85	0.69	0.77
80%	2.24	0.72	0.85
90%	2.95	0.94	1.22
Observations	1068	1068	872

Table 6. RMSE by recovery rate and hazard rate. All firms considered.

Table 7 displays RMSE by firm and hazard rate, using a 50% recovery rate. Linear and quadratic hazard rates achieve better fit than the constant hazard rate for all firms considered.

We conclude that linear and quadratic hazard rates accomplish smaller RMSE values than the constant hazard function, and that the choice of the recovery rate does not introduce large differences, as long as it lies below 80%.

Figure (2) represents the implied survival probabilities by recovery rate for the quadratic hazard rate. The average implied survival probability decreases as recovery rate increases.[26] The reason is that, as the recovery rate increases, other things equal, bond prices would increase. However, in our case, it is bond prices what are given. Therefore, to fit those prices, a higher recovery rate requires a lower survival probability.

RMSE should be smaller, although Table 6 does not show this to be so. The reason is that, for Caja Madrid and Telefonica, there exist dates for which we have not estimated quadratic hazard rate parameters because we only have three bonds traded. In these cases, as we have mentioned, we only estimate the model for constant and linear hazard rates. The fact that in those dates the RMSE are too small for the linear hazard rate (because we are fitting three bonds using two parameters) implies that the annual average RMSE for the linear hazard function appears smaller than the RMSE for the quadratic hazard rate.

[26] This result holds when considering separately each firm, and also for constant linear and hazard rates.

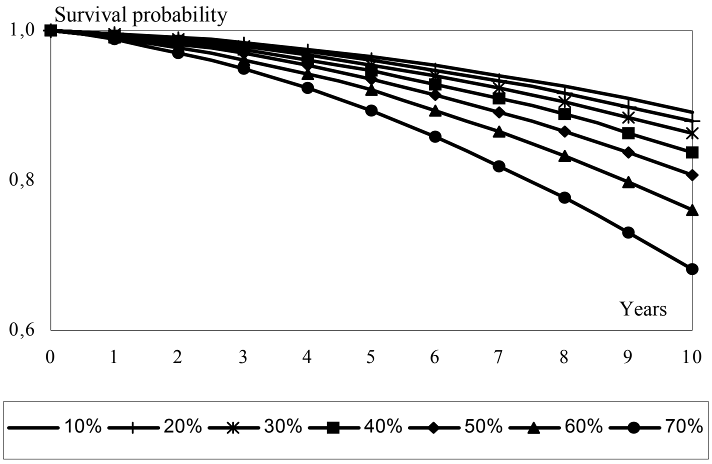

Figure 2: Average survival probabilities, by recovery rate, as a function of years to maturity. All firms have been considered.

	Constant	Linear	Quadratic
BBVA	2.77 (208)	1.77 (208)	2.22 (208)
Caja Madrid	4.17 (116)	0.10 (116)	0.12 (35)
Endesa	1.60 (218)	0.20 (218)	0.21 (218)
Repsol YPF	3.22 (89)	0.53 (89)	0.31 (89)
SCH	1.38 (217)	0.86 (217)	0.40 (217)
Telefonica	1.15 (220)	0.32 (220)	0.27 (105)

Table 7. RMSE by firm and hazard function. 50% recovery rate. Number of observations in brackets.

4.2 Comparing model and market CDS premiums

The pricing error is defined as the market premium minus the model premium, and it is summarized by the Mean Pricing Error (MPE) and the Mean Absolute Pricing Error (MAPE), both expressed in basis points. MAPE is used as the model's pricing quality and MPE is used to analyze whether the model is, on average, overestimating (negative) or underestimating (positive) the firm's credit quality.

4.2.1 Choosing the best specification

Table 8 shows MPE and MAPE by hazard rate and firm for a 50% recovery rate and considering all CDS maturities. The linear hazard rate generates, on average, the smallest MAPE, followed closely by the quadratic hazard rate.

By firms, the sign of MPE varies with the hazard rate used, except in the case of Endesa and Repsol YPF, where the model overestimates (Endesa) and underestimates (Repsol YPF) the firm's credit risk.

Considering all maturities and a 50% recovery rate, the specifications which best price each firm's CDS are the constant hazard rate for SCH, the linear hazard rate for BBVA, Caja Madrid, Endesa and Telefonica and the quadratic hazard rate for Repsol YPF. Considering only these specifications, BBVA is the firm best priced by the model, followed by a second group formed by Endesa, Caja Madrid and Telefonica, with SCH and Repsol YPF being the firms worst priced. These three groups can also be observed, but in reverse order, in terms of the level of CDS premiums, indicating that the model's ability to price CDS is inversely related to the level of the firm's CDS quotes.

	Constant	Linear	Quadratic
BBVA	-40.29	**2.15**	0.89
	(40.37)	**(6.88)**	(10.53)
Caja Madrid	-33.28	**12.35**	12.35
	(33.74)	**(12.47)**	(12.75)
Endesa	-38.42	**-4.48**	-6.39
	(38.95)	**(7.71)**	(8.99)
Repsol YPF	98.90	49.07	**45.16**
	(104.60)	(55.88)	**(54.14)**
SCH	**-19.42**	16.95	19.04
	(29.53)	(33.32)	(34.24)
Telefonica	-2.44	**15.04**	-5.50
	(24.93)	**(20.45)**	(26.79)
Total	-15.47	11.45	7.81
	(39.33)	(20.00)	(22.57)

Table 8. MPE (MAPE) in basis points by hazard rate and firm. 50% recovery rate. Cells in bold characters identify the hazard rate which best prices each firm's CDS.

To get an idea of the model's performance to price CDS, Figures (3) to (8) show model and market CDS premiums using, for each firm, the hazard rate which generates the smallest MAPE.[27] For all firms, model implied CDS premiums are more volatile than market premiums.

[27] The scale which measures the CDS premiums in all subplots is the same, from 0 to 140 basis points, except for Repsol YPF, in which it ranges from 0 to 450 basis points.

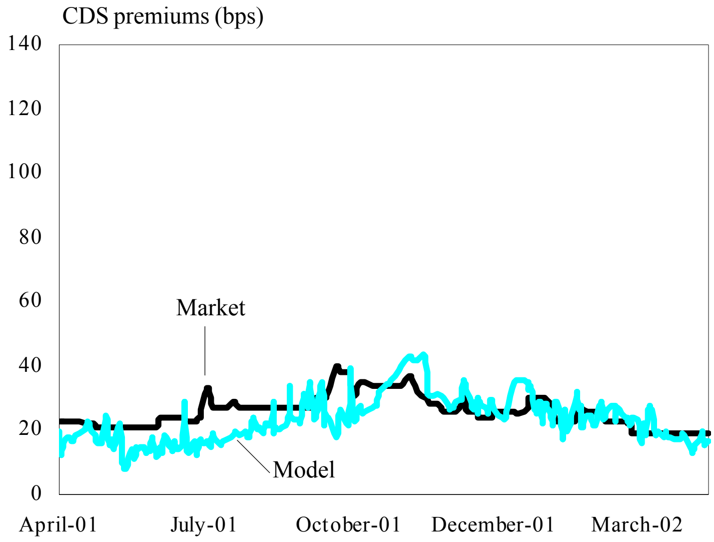

Figure 3: **BBVA**'s market and model implied five years CDS premiums, in basis points, using a linear hazard rate and a 50% recovery rate.

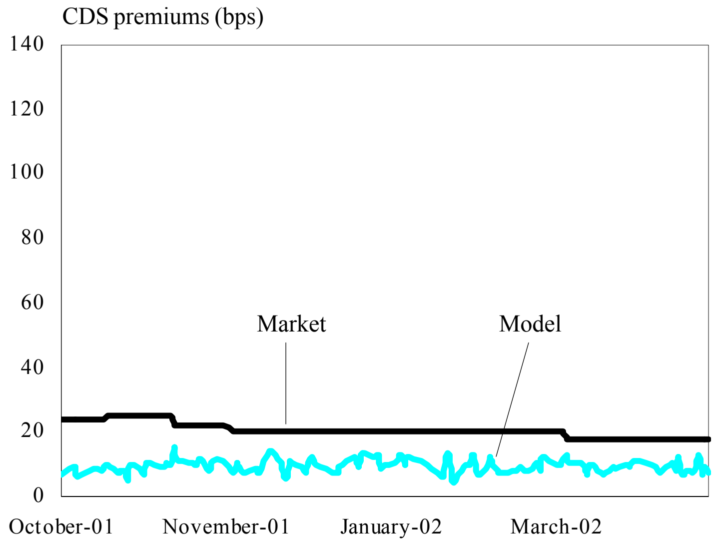

Figure 4: **Caja Madrid**'s market and model implied five years CDS premiums, in basis points, using a linear hazard rate and a 50% recovery rate.

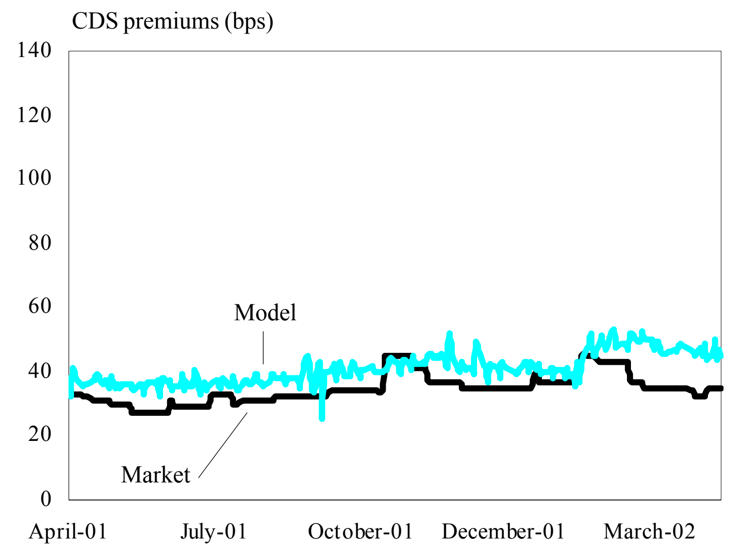

Figure 5: **Endesa**'s market and model implied five years CDS premiums, in basis points, using a linear hazard rate and a 50% recovery rate.

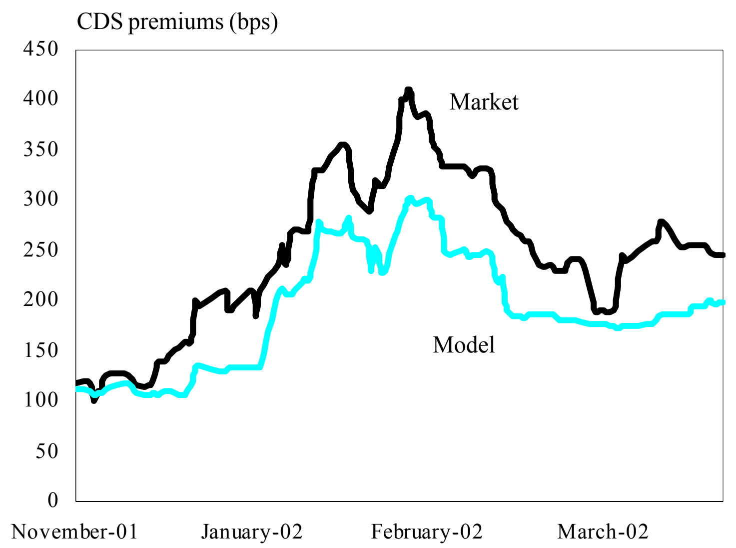

Figure 6: **Repsol**'s market and model implied five years CDS premiums, in basis points, using a linear hazard rate and a 50% recovery rate.

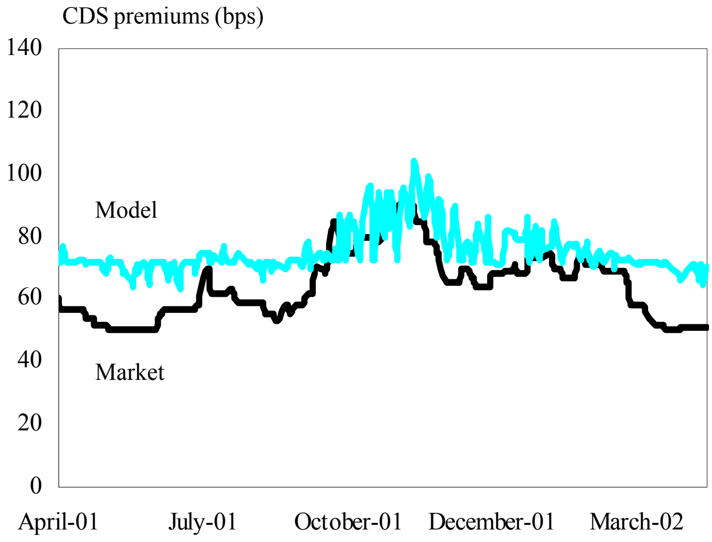

Figure 7: **SCH**'s market and model implied five years CDS premiums, in basis points, using a constant hazard rate and a 50% recovery rate.

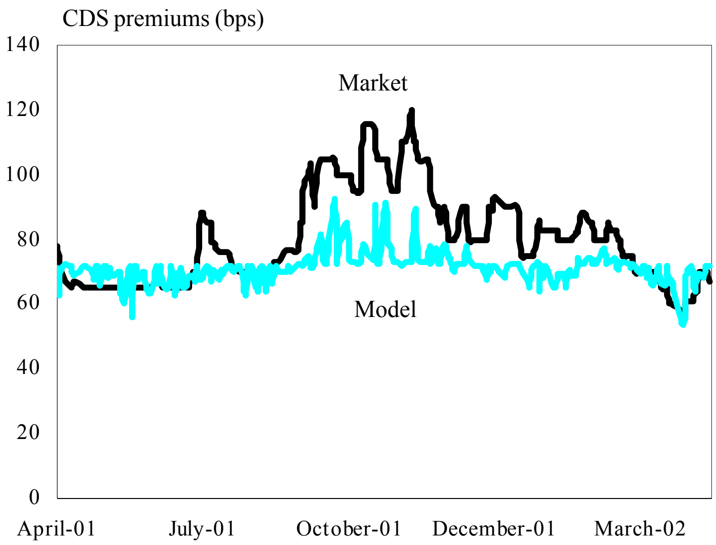

Figure 8: **Telefonica**'s market and model implied five years CDS premiums, in basis points, using a constant hazard rate and a 50% recovery rate.

4.2.2 CDS premium sensitivity to the recovery rate

The same recovery rate is used when estimating hazard rate parameters from bond market prices and when pricing CDS using the estimated parameters. As shown in Figure (2) survival probabilities estimated from bond prices are a decreasing function of recovery rates. In a CDS, a higher recovery rate reduces the premium because it reduces the payment of the protection seller in case of default. At the same time, a lower survival probability increases the premium because it increases the probability of the reference entity defaulting. Therefore, the effect of the choice of the recovery rate on the survival probabilities derived from bond prices, might be partially offset when pricing CDS. The question which remains is: in which degree is that effect offset?.

Table 9 shows MPE by hazard rate and recovery rate. For all hazard rates considered, except for the constant hazard function, MPE remain approximately constant when varying the recovery rate in the interval from 20% to 60%.

	20%	30%	40%	50%	60%	70%	Observations
Constant	-27.40	-24.78	-21.63	-15.47	-4.59	7.89	1068
Linear	10.92	11.12	11.38	11.45	11.77	13.11	1068
Quadratic	7.76	7.69	7.74	7.81	8.25	8.47	872

Table 9. MPE (basis points) by hazard rate and recovery rate. All firms considered.

Table 10 shows MPE by CDS maturity and recovery rate using a linear hazard rate. For all maturities MPE increases as maturity grows, and although MPE is not constant when recovery rate varies, its variation in the interval from 20% to 60% is generally less than 0.5 basis points per each 10% variation in the recovery rate. The conclusions are similar for quadratic hazard rate, whereas MPE are not so insensitive to the recovery rate chosen for the constant hazard function.

	20%	30%	40%	50%	60%	70%
1 year	-0.36	0.15	0.69	0.67	1.11	4.28
2 years	5.60	6.01	6.44	6.39	6.70	9.20
3 years	5.18	5.49	5.82	5.76	5.97	7.86
4 years	9.35	9.58	9.83	9.76	9.91	11.29
5 years	11.19	11.35	11.54	11.49	11.62	12.61
6 years	12.97	13.07	13.22	13.22	13.82	14.11
7 years	14.23	14.30	14.44	14.51	14.74	15.37
8 years	15.94	16.00	16.14	16.31	16.65	17.24
9 years	17.15	17.22	17.40	17.69	18.19	18.87
10 years	17.97	18.06	18.30	18.73	19.41	20.28

Table 10. MPE (basis points) by maturity and recovery rate. Linear hazard rate. All firms considered.

Table 11 shows MPE by firm and recovery rate using a linear hazard rate. As before, except when we move to a 70% recovery rate, MPE values seem relatively insensitive to the choice of the recovery rate, varying around two basis points when moving from 20% to 60%. In the case of Repsol YPF, these variations are higher, but if we consider the magnitude of MPE compared with the other firms, the relative change in MPE is small. Comparing the results for the three hazard rates, it is apparent that while MPE show a clear insensitivity to movements in the recovery rate for both the linear hazard rate and the quadratic hazard rate, the constant hazard rate does not exhibit such feature for all firms.

	20%	30%	40%	50%	60%	70%	Observations
BBVA	3.11	3.14	2.79	2.15	1.29	0.09	208
Caja Madrid	12.17	12.25	12.25	12.35	12.38	12.58	116
Endesa	-5.36	-5.26	-5.08	-4.84	-4.47	-4.04	218
Repsol YPF	46.30	46.94	47.94	49.07	50.86	53.97	89
SCH	16.67	17.02	17.39	16.95	16.88	20.69	217
Telefonica	13.61	13.82	14.43	15.04	16.31	18.32	220

Table 11. MPE (basis points) by firm and recovery rate. Linear hazard rate.

To sum up, for the majority of the cases studied, it seems that the proposed model to price CDS generates CDS premiums which are relatively insensitive to the choice of the recovery rate, as long as it lies between 20% and 60% for linear and quadratic hazard functions, but not for the constant hazard function.

4.2.3 Analyzing pricing errors

In order to analyze how differences between market and model premiums are related to firms, hazard rates, recovery rates and CDS maturities and dates, we regress MAPE on dummy variables for firm, CDS maturity, recovery rate and period.

The following recovery rates are considered: 20%, 30%, 40%, 50%, 60% and 70%. Period dummies consist in four groups: May 01/July 01, August 01/October 01, November 01/January 02, and February 02/April 02.

Table 12 shows the regression results for the linear hazard process; the results for the constant and quadratic hazard functions are similar.[28]

Most parameters are statistically different from zero, and the R^2 value is 39.52%. Mispricings strongly differ between firms, Repsol YPF being the firm with highest errors, followed by Telefonica and SCH in the second group, and finally BBVA, Endesa and Caja Madrid, which are the firms best priced by the models. On average, firms with higher CDS premiums, are the worse priced.

The maturity of the CDS contract is also indicative of the pricing error, since the coefficients of the maturity dummies are significant (except for four and five years) and with the smaller errors in the interval from four to seven years. With respect to the recovery rates, we observe that the differences in their coefficients are insignificant, and they are not significantly different from zero, corroborating the fact that the choice of the recovery rate is not a crucial fact in the pricing of CDS, whenever it lies in a reasonable interval. Finally, the parameter estimates for the contract's quote date dummies show that pricing errors in 2002 were smaller than in 2001. The results for constant and quadratic hazard functions share the conclusions implied for the linear hazard function.

[28] Since each set of dummies contains mutually exclusive categories, their values sum to one for each observation. The coefficient of one of the dummies is set to zero as the identifying restriction.

$R^2 = 39.52$		Coefficient	Std. Err.	p-value
	Constant	20.69	0.41	0.00
Firm	BBVA	-9.06	0.29	0.00
	Caja Madrid	0		
	Endesa	-8.07	0.29	0.00
	Repsol YPF	43.73	0.35	0.00
	SCH	16.94	0.29	0.00
	Telefonica	4.71	0.29	0.00
CDS maturity	1 year	8.47	0.33	0.00
	2 years	4.46	0.33	0.00
	3 years	2.49	0.33	0.00
	4 years	0.62	0.33	0.06
	5 years	-0.11	0.33	0.73
	6 years	0		
	7 years	0.77	0.33	0.02
	8 years	2.33	0.33	0.00
	9 years	3.94	0.33	0.00
	10 years	5.37	0.33	0.00
Recovery rate	20%	-0.22	0.26	0.38
	30%	-0.21	0.26	0.40
	40%	-0.18	0.26	0.47
	50%	0		
	60%	-0.07	0.26	0.77
	70%	0.60	0.26	0.02
Date	May 01/Jul. 01	0		
	Aug. 01/Oct. 01	-9.13	0.22	0.00
	Nov. 01/Jan. 02	-12.51	0.22	0.00
	Feb. 02/Apr. 02	-10.11	0.22	0.00

Table 12. Pricing error analysis. The table shows the results of regressing mean absolute pricing errors (MAPE) with respect to dummy variables for firm, CDS maturity, recovery rate and period. A linear hazard rate specification is used.

5 Concluding remarks

We have presented a simple model to price CDS using the information contained in bond prices. Judging from the model implied CDS premiums shown in Figures (3) to (8) and the implied pricing errors, we can conclude that the model does a pretty good job in terms of CDS pricing, taking into account all the simplifying assumptions. The model is almost no time consuming and, as such, it can give an investor a very good (and quick) idea of the level of CDS premiums as a first step to, if model and market prices present significant differences, apply a more complex model.

In line with results in previous literature, the model implied CDS premiums are relatively insensitive to the choice of the recovery rate, as long as it lies in a realistic interval. Model implied CDS premiums are clearly more volatile than market premiums. This is likely due to the scarcity of traded bonds from which the model extracts, each day, the parameters which describe survival probabilities. Small variations in the estimated parameters have significant effects on the survival probabilities and hence in the model CDS premiums. Additionally, the model has been shown to work better the lower the firm's credit risk.

Any attempt to improve the performance of the model should address the simplifying assumptions used here. First, the spread between default-free and corporate debt is assumed to be due exclusively to credit risk, ignoring other factors such as liquidity risk. However, the spread between risk free debt and firm debt might be affected by several factors including, but not limited to, credit risk. In order to extract the true probabilities of default from the market price of a bond, all the value unrelated to the credit risk should be stripped from its market price.

Second, protection seller default risk has been ruled out. As Hull and White (2001) explains, this assumption can be maintained without influencing the results when the correlation between the credit quality of the protection seller and the credit quality of the reference entity is zero. For considering protection seller default risk and the correlation between the protection seller and reference entity credit qualities, the model should include hazard processes for both of them and a way to take into account the correlation. Moreover, one should also take into account the possibility of the protection buyer defaulting during the life of the contract, as well as the credit risk correlation between the protection seller and the protection buyer and between the protection buyer and the reference entity.

Third, the assumption of a constant recovery rate is also problematic because, as shown by Altman et al. (2005), there exists a strong negative correlation between default probabilities and recovery rates.

Finally, CDS contracts typically imply physical delivery in case of default, but here they have been priced under cash settlement. If a credit event occurs, the protection buyer in a CDS specified with physical settlement has the ability to deliver one of a large number of deliverable assets into the contract. The wider the spectrum of deliverable assets, the more value the delivery option will have. In order to value a CDS considering physical settlement, the delivery option should be valued. However, it is very difficult to quantify the value of the delivery option in a CDS. We are not aware of any paper estimating the delivery option in CDS.

Any step forward in relaxing those assumptions would shed more light about the determinants of CDS market prices.

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