Equity-credit modelling
Where are we. Where should we go

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Contents

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1 Introduction

After more than a decade of continuous growth ignoring what was going on in other markets and products, credit risk models face the need to build up stronger (than existing) links with modelling approaches for other asset classes, in particular with those for equity and equity derivatives pricing. This need has its main causes in the growing number of empirical studies showing strong links between the evolution of credit, equity and business cycles. The traditional segregation of equity and credit research facilities in investment banks has been questioned in light of such findings.

We analyze the extension of established credit risk models for the joint modelling of equity and credit products.[1] As we explain below, structural and reduced form models represent the two most important competing approaches in credit risk modelling. The main distinguishing characteristic of structural models is the link they provide between the firms' fundamental financial variables (assets and liabilities) and default probabilities. Although easier to calibrate, reduced form models lack a direct link between credit risk and the information regarding the firms' financial situation incorporated in their assets and liabilities structure.

In light of the analysis we present, structural models seem more suitable than reduced models to provide the basis for general equity-credit models. However, they need to be extended both in their theoretical and empirical sides. Theoretically, they need to include the possibility of default due to liquidity shortages and high financing costs. Empirically, they need to include more information about the firms' future cash flows and short term debt repayments. For the success of such extensions, in particular the information needs, we argue in favor of merging credit risk and fundamental (equity) analysis departments at investment banks, in order to make the most of the information gathered by both divisions. The proposed extensions would provide structural models with the ability to price credit and equity instruments.

[1] Note that there are alternative routes to develop such a joint modelling framework: (i) extending existing models used for equity pricing, and (ii) developing new models altogether.

This Section contains a quick review of structural and reduced form models. Section 2 presents an empirical exercise using a reduced form model which shows the strong links between equity and credit markets. Sections 3 and 4 study the ability of structural and reduced form models to form the basis of a joint equity-credit pricing model.

1.1 Structural vs. reduced credit models

There are two primary types of models in the literature that attempt to describe default processes for defaultable instruments, usually referred to as *structural* and *reduced-form* models. Elizalde (2005a, b, c) reviews the existing academic literature.

Structural models use the evolution of firms' structural variables, such as asset and debt values, to determine the time of default. Merton's model (1974) was the first modern model of default and is considered the first structural model. In Merton's model, a firm defaults if, at the time of servicing the debt, its assets are below its outstanding debt. A second approach, within the structural framework, was introduced by Black and Cox (1976). In this approach default occurs as soon as the firm's asset value falls below a certain threshold. In contrast to the Merton approach, default can occur at any time.

Reduced form models do not consider the relation between default and firm value in an explicit manner.[2] In contrast to structural models, the time of default in intensity models is not determined via the value of the firm, but it is the first jump of an exogenously given jump process whose intensity represents the instantaneous default probability. Reduced form models use market prices of the firms' defaultable instruments (such as bonds or credit default swaps) to extract both their default probabilities and their credit risk dependencies. They rely on the market as the only source of information regarding the firms' credit structure (without considering any information included in balance sheets or equity prices).

[2] Intensity models represent the most extended type of reduced form models. Brody, Hughston and Macrina (2005) present a different reduced form approach, based on the amount and precision of the information received by market participants about the firm's credit risk.

Structural models provide a link between the credit quality of a firm and the firm's economic and financial conditions. Thus, defaults are endogenously generated within the model instead of exogenously given as in the reduced approach. Another difference between the two approaches refers to the treatment of recovery rates: whereas reduced models exogenously specify recovery rates, in structural models the value of the firm's assets and liabilities at default will determine recovery rates.

2 Reduced form models: assessing the magnitude of the problem

How important is the correlation between equity and credit markets? Should we be worrying about jointly modelling equity and credit risk?

This Section summarizes the results in Elizalde (2005d). The paper estimates an intensity model for 14 US firms, decomposes their credit risk in several risk factors, computes the importance of such factors, and relates them with equity indexes (S&P 500 and Dow Jones).

Let λ_t denote the default intensity (or instantaneous default probability) of a firm at time t and L_t the expected loss rate in the market value of a firm's bond if it defaults at t. Duffie and Singleton (1999) show that a defaultable bond can be priced as if it was a default-free bond by replacing the usual short-term interest rate process r_t with a default adjusted short rate process $r_t + \lambda_t L_t$.

Reduced form models are not able to separately estimate (using bond data) the part coming from default risk λ_t from the part coming from the loss given default L_t. A common practice is to assume an exogenously given process for the recovery rate, generally a constant and deterministic rate based on historical data. We prefer to estimate $s_t = \lambda_t L_t$ directly, where s_t contains the firm's credit risk (probability and severity of default).

We express the different firms' credit spreads $s_{1,t}, ..., s_{J,t}$ as linear functions of one common unobservable risk factor Z_t plus a firm idiosyncratic error term $\varepsilon_{j,t}$:

$$s_{j,t} = b_j Z_t + \varepsilon_{j,t}, \tag{1}$$

for each firm $j = 1, ..., J$.

Z_t is an unobservable factor which affects the credit spreads $s_{j,t}$ of *all* firms in the sample. The impact of this common factor differs across firms, and is given by the parameter b_j for each firm j. $\varepsilon_{j,t}$ is different for each firm. We are interested on how much credit risk we can explain with the common factor Z_t. Even though the terms $\varepsilon_{j,t}$ may be correlated across firms, we just want to know how much credit risk correlation can we pick up with the simplest possible correlation structure: a single common factor.

Using affine processes for the unobservable risk factors, one can obtain closed form solutions for bond prices. Using bond data for the different firms during the sample period and Vasicek processes for the dynamics of the unobservable risk factors, the realization of each risk factor and their impact on the firms' credit risk are estimated using a Maximum Likelihood procedure based on Kalman Filters.

The dataset consists of daily data covering the period from July 2001 to November 2003 for 14 different US firms. The main result is the following (Table 1): with just one common factor Z_t (affecting all firms in the sample) the model explains between 15 and 91% of the firms' credit risk, with an average across firms of 68%. That single credit risk factor is able to explain, except in two cases, more than 50% of the firms' credit risk.

Firm	%	Firm	%
Owens	80.59	Marriott	15.25
CSC	82.70	Norfolk	37.78
Comcast	64.16	Sears R.	72.12
W. Disney	78.87	TCI	57.93
Ford Credit	87.13	Warner	61.57
GM	91.26	USX	77.11
Hertz	88.73	Union P.	56.45

Table 1: Average (across bonds and time) percentage of the firms' credit risk explained by the common factor Z_t. Mean (Std. Dev.) across firms: 68.02% (21.37%).

In order to better understand what is behind the common risk factor Z_t affecting the credit risk of all firms, it is helpful to look at some observable variables related with the level of credit risk and economic activity in the US during the period analyzed. It can be shown that the evolution of credit risk factor Z_t resembles that of several variables directly related with credit risk levels in the US economy: BBB-rated credit spreads over government bond yields, profit warnings, number of defaults, downgrades as a percentage of all rating actions, and KMV default probabilities for the US economy. Therefore, the common risk factor explaining most of the credit risk of the firms in our sample seems to be common to all firms in the US economy, which suggests that the results in this paper can probably be extrapolated to other US corporates.

The initial objective of the paper was to investigate the importance of credit risk correlations. In the model proposed above, credit risk correlations arise because the common factor Z_t affects all firms' credit risk. Therefore, the strategy was to estimate the realization of the common factor Z_t, the factor loadings b_j, and determine how much of the firms' credit risk can Z_t explain by itself. It turned out that, as we showed above, Z_t explains (on average across firms) around 68% of their credit risk, quite a big number. However, the relevance of this study to our purposes is the relationship between Z_t, the main driver of credit risk, and equity. Since Z_t represents a common global factor, the importance of credit-equity dependence is examined by comparing Z_t with global equity indicators of the US economy.

Figure (1) presents the evolution of the common risk factor Z_t together with that of the two major stock indexes in the US: S&P 500 and Dow Jones. The dynamics of Z_t are inversely related to those of the S&P 500 and Dow Jones, with correlation coefficients of -0.76 and -0.77 respectively.

To sum up, the results show that a high part of the firms' credit risk is common to all of them and it is strongly negatively related to global equity indexes. Therefore, the answer to the question we posed at the beginning of this section is: **Yes**, we should be worrying about jointly modelling equity and credit risk.

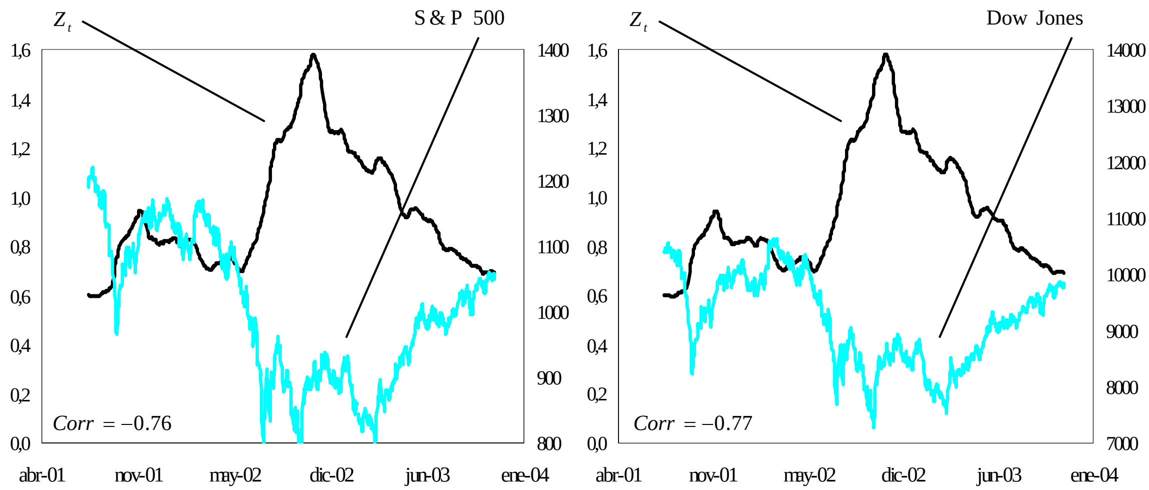

Figure 1: Evolution of the common risk factor Z_t, S&P 500 and Dow Jones. Source: Yahoo finance (close prices adjusted for dividends and splits).

After showing the strong links between credit and equity markets (which shouldn't be surprising), let's try go gauge the ability of current credit risk models to capture such relationship. But before moving on, it is interesting to think about the roots of such relationship which, we argue, lie (as almost everything) in **cash**. The cash generated by the firm's activities is usually called cash flow, but firms also can get (costly) cash by external funding. The market value of equity comes from future dividend payments, which come from cash flows. Credit risk measures the ability of firms to pay their liabilities, which are paid with cash.

Cash flows are the basis of equity-credit modelling and most of what follows deals with extending structural credit risk models to accommodate cash flows, both theoretically and empirically.

3 Structural models: a direct way for a unified equity-credit modelling

Given the two frameworks for credit risk modelling, structural and reduced form models, it seems convenient to gauge their respective ability to be extended in order to accommodate equity pricing. We focus in this Section in structural models.

3.1 State of the art (theory)

The differentiating characteristic of structural models is their reliance on the variables determining the firms' fundamental financial situation, such as the value of assets and liabilities, to explain their credit risk. Imagine, for simplicity, a firm with the following (market value) balance sheet at time t:

Assets	Liabilities
V_t	Equity E_t
	Debt D_t

(2)

Merton (1974) makes use of the Black and Scholes (1973) option pricing model to value corporate liabilities. This is an straightforward application if we adapt the firm's capital structure and the default assumptions to the requirements of the Black-Scholes model. Let us assume that the capital structure of the firm is comprised by equity and by a zero-coupon bond with maturity T and face value of K, whose values at time t are denoted by E_t and D_t respectively, for $0 \leq t \leq T$. The firm's asset value V_t is simply the sum of equity and debt values. Under these assumptions, equity represents a call option on the firm's assets with maturity T and strike price of K. If at maturity T the firm's asset value V_T is enough to pay back the face value of the debt K, the firm does not default and the shareholders receive $V_T - K$. Otherwise ($V_T < K$) the firm defaults, bondholders take control of the firm, and the shareholders receive nothing. Implicit in this argument is the fact that the firm can only default at time T. This assumption is important to be able to treat the firm's equity as a

European call option, and therefore apply the Black-Scholes pricing formula.[3]

The firm's asset value is assumed to follow a diffusion process given by

$$dV_t = rV_t dt + \sigma_V V_t dW_t, \tag{3}$$

where σ_V is the (relative) asset volatility and W_t is a Brownian motion.

Applying the Black-Scholes pricing formula, the value of the equity at time t $(0 \leq t \leq T)$ is given by

$$E_t\left(V_t, \sigma_V, T-t\right) = e^{-r(T-t)}\left[e^{r(T-t)} V_t N\left(d_1\right) - KN\left(d_2\right)\right] \tag{4}$$

where $N\left(.\right)$ is the distribution function of a standard normal random variable and the expressions for d_1 and d_2 are given by

$$d_1 = \frac{\ln\left(\frac{e^{r(T-t)}V_t}{D}\right) + \frac{1}{2}\sigma_V^2\left(T-t\right)}{\sigma_V\sqrt{T-t}}, \tag{5}$$

$$d_2 = d_1 - \sigma_V\sqrt{T-t}. \tag{6}$$

Therefore, the value of the debt at time t will be given by $D_t = V_t - E_t$. The probability of default at time T is given by

$$P\left[V_T < D\right] = N\left(-d_2\right). \tag{7}$$

In order to implement Merton's model we have to estimate the firm's asset value V_t, its volatility σ_V (both unobservable processes), and we have to transform the debt structure of the firm into a zero-coupon bond with maturity T and face value K.

One important problem of Merton's model is the restriction of default time to the maturity of the debt, ruling out the possibility of an early default, no matter what happens with the firm's value before the maturity of the debt. If the firm's value falls down to minimal levels before the maturity of the debt but it is able to recover

[3]The rest of assumptions Merton (1974) adopts are the inexistence of transaction costs, bankruptcy costs, taxes or problems with indivisivilities of assets; continuous time trading; unrestricted borrowing and lending at a constant interest rate r; no restrictions on the short selling of the assets; the value of the firm is invariant under changes in its capital structure (Modigliani-Miller Theorem) and that the firm's asset value follows a diffusion process.

and meet the debt's payment at maturity, the default would be avoided in Merton's approach.

The paper by Black and Cox (1976) is the first of the so-called *First Passage Models* (FPM). First passage models specify default as the first time the firm's asset value V_t hits a lower barrier K_t, allowing default to take place at any time. FPM have been extended to account for stochastic interest rates, bankruptcy costs, taxes, debt subordination, strategic/endogenous default (the default barrier K_t is optimally chosen by equityholders), time dependent and stochastic default barrier, jumps in the asset value process, etc. Although these extensions introduce more realism into the model, they increment the analytical complexity.[4]

In all cases, the value of equity E_t, debt D_t, and default probabilities can be expressed as a function of the model parameters (asset value V_t, asset volatility σ_V, default threshold K_t, ...).

Theoretically, structural models provide a satisfactory framework for jointly modelling equity and credit risks because they provide formulas for equity, debt and default probabilities. However, they were introduced to price defaultable instruments using equity prices as given, and not to jointly price both types of assets. In fact, to the best of our knowledge, there is not a single academic paper which uses a structural model to price both equity and defaultable claims.

3.2 State of the art (empirics): Equity and balance sheet inputs for credit models

Almost all proposed ways to empirically implement structural models follow a simple two step procedure:[5]

1. The model parameters are estimated using the theoretical formulas for the value of equity implied by a given model, equity prices and balance sheet information

[4] For an extensive review of FPM see Bielecki and Rutkowski (2002, Chapter 3) and references therein.

[5] Elizalde (2005b) provides a detailed literature review of structural models estimation.

about the firms' liabilities.

This first step is not straightforward. Given an estimation of the default threshold K_t, which in itself is a major task, the literature provides several ways of estimating the unknown asset value process V_t and its volatility σ_V :

(a) The first method makes use of Itô's Lemma to obtain a system of two equations in which the only two unknown variables are V_t and σ_V.

(b) Duan (1994) points out some drawbacks of the previous method and proposes an alternative one, based on maximum likelihood estimation using equity prices and the one-to-one relationship between equity and asset levels given by (4). Duan and Fulop (2005) extend Duan's (1994) maximum likelihood estimation method to account for the fact that observed equity prices might be contaminated by trading noises.

(c) A different way of estimating V_t and σ_V, which can be found on Jones et al. (1984), consists simply on estimating the asset value as the sum of the equity market value, the market value of traded debt and the estimated value of non-traded debt. Provided with a time series for V_t we can estimate its volatility σ_V.

(d) Hull, Nelken and White (2003) propose a way to estimate the model's parameters from implied volatilities of options on the company's equity, avoiding to estimate σ_E and to transform the firm's debt structure into a zero-coupon bond.

2. Use the parameter estimates in the previous step and the theoretical formulas for bond prices to price them, as well as to compute default probabilities and to price other defaultable instruments.

Note that this approach (i) uses equity and balance sheet information as inputs in order to (ii) price defaultable instruments. It represents the initial aim of structural models: pricing defaultable instruments using information about the firm fundamental variables.

The empirical performance of these models when pricing defaultable instruments can not be considered satisfactory, mainly because they tend to underpredict in a considerable degree the credit spreads priced in the market of defaultable instruments (see Eom, Helwege and Huang, 2003, and Huang and Huang, 2003).

3.3 New empirical developments: Equity and credit inputs for credit models

Fortunately, and besides any theoretical improvements, structural models are evolving in order to make use of a higher amount of information in their calibration.

Bruche (2005) proposes a simulated likelihood method to estimate structural models which **not only uses equity and balance sheet information to calibrate the model** as before, **but it also uses market prices of defaultable instruments** (bonds, CDSs, ...).

Although structural models provide formulas for both debt and equity prices, the strategy of previous empirical papers was to use only equity prices and formulas to estimate the model parameters, and use them to price debt. But this way we are throwing away all information contained in debt prices when calibrating the model. That's not efficient, is it?

Both equity and defaultable instrument prices depend on a common factor, the firms' unobservable asset value V_t, and the model parameters. Using a state space model and given dynamics for V_t (for example lognormal in the Merton model) a maximum likelihood procedure would allow to recover both the realization of V_t and the model parameters.

Therefore, Bruche (2005) extends the estimation of structural models to allow them to use as much information as possible: accounting information, equity prices, bond prices, CDS prices, and any other existing asset for whom a theoretical formula can be derived within structural models.

Notice the difference between previous empirical works (using equity prices to estimate the model and then price defaultable instruments) and Bruche's approach (using equity and defaultable prices to estimate the model and then price defaultable instruments). We are progressing, but ... why do we only use the model to price defaultable instruments? Any reliable equity-credit model should be able, not only to use equity and credit information, but also to price equity and credit products; something still missing in the literature. Nobody will take seriously any equity-credit modelling unless the modeler feels comfortable enough as to use it to price both asset types.

3.4 What's next? Equity and credit inputs for equity-credit joint models

So far, we have structural models which use information from equity and credit products but which are only used to price credit products. As we argued above, it is fairly clear that to become a reliable basis for equity-credit modelling, structural models need to be further extended to be able to price both, equity and credit products.

Such extensions include both empirical and theoretical work, which, as we claim next, can only come together.

3.4.1 Empirical findings which call for theoretical extensions

Existing structural models only consider one type of default trigger: a small enough asset value where the asset value represents the discounted future expected cash flows. However, a firm with good future prospects (and therefore high asset value) might experience a situation of (i) low current cash flows which are not enough to pay current debt payments and (ii) high financing costs which difficult raising new funds. Such situation can trigger the default of the firm, besides a high asset value. Existing structural models do not consider this cause of default.

The role of cash shortages and financing costs on default triggering

Davydenko (2005) analyzes whether defaulted firms have either low asset value or liquidity shortages (or both). The paper uses a sample of US (speculative rating-grade) defaulted and non-defaulted bond issuers from 1996 to 2003. The author presents evidence suggesting that viewing default as being triggered by asset value crossing a particular threshold is not consistent with the data, the reason being that there is a significant fraction of defaulted firms with high asset values and low liquidity. The interaction between liquidity shortages and high funding costs is found to be a significant predictor of default:

> (p. 5) "... even incorporating both value- and liquidity-based defaults does not ensure accuracy if the costs of external financing for a particular firm at a particular time are not simultaneously taken into account. ... Overall, the apparent inability of structural models to predict default probabilities and spreads especially over short horizons may not be surprising, given that their simplifying assumptions regarding the default trigger are not fully supported by the data."

The author shows that the combination of liquidity shortages and high external financing costs is as important as low asset values in explaining firms' defaults. The consequence of the previous finding for the empirical performance of those structural models which only consider value-based defaults (and we are not aware of any which lies outside this category) is clear: forgetting one of the main default triggers (liquidity shortages plus external financing costs) makes the models underestimate default probabilities, which is the biggest sin attributed to structural models so far. Although the literature has proposed different ways to correct this problem (see Elizalde 2005b), they lack the empirical backing that Davydenko provides for the role of liquidity shortages and high external financing costs.

Implications for modelling

Davydenko (2005) presents evidence for the need to extend structural models to include not only asset-based defaults (when the asset value V_t goes below the default threshold K_t), but also liquidity-based defaults (when the firm experiences a liquidity shortage and external fund are so expensive that it can not satisfy its contracted payments). This extension would probably involve modelling both the process for the cash flow and for the cost of external financing, and any dependence between the two. It is not the objective of this report to speculate further on this modelling issue but just to point out the need for it.

Introducing cash flow modelling into structural models would not only improve their empirical performance, but it will provide the perfect seed to use them to price equity and equity-related products. Therefore, the way one would like to introduce cash flow modelling into structural models should resemble the way it is used in equity models, or at least in a way that it is compatible with such models.

Leland (2005), in his comments on Davydenko's results, argues that even though it is true that exogenous default models (i.e. the ones in which the default threshold K_t is given exogenously) do not include liquidity-based defaults, endogenous default models do. In endogenous default models equityholders choose the default threshold endogenously and, therefore, take into account things like the possibility of future liquidity shortages. However, when Leland (2004) compares the empirical performance of exogenous vs. endogenous default models, he finds that both of them underpredict historical default probabilities, particularly short-term ones. So it seems endogenous default models still miss something; probably they are not able to completely capture liquidity-based defaults.

Davydenko (2005) refers to various theoretical papers extending structural models to account for defaults due to either liquidity shortages (Anderson and Sundaresan 1996) or both liquidity shortages and low asset values (Fan and Sundaresan 2000). However interesting such extensions might be in their theoretical insights, they are quite difficult to be empirically implemented in a structural model aimed at pricing both equity and debt instruments. In fact, to the best of our knowledge, they have not been so. Nevertheless, they represent, in our opinion, the right direction for future research (although this research should be mainly driven by empirical concerns regarding its implementation capabilities).

The arguments put forward above have already been taken into account by the industry. In particular, the need to take into account the role of liquidity shortages to explain defaults. Jordão and Stein (2005) use a liquidity-based model, based on Wilcox (1971), to predict EBITDA volatility. The (binomial) model is shown to perform satisfactorily when predicting EBITDA volatility but, as the authors recognize, it does not have enough structure to be used as a joint equity-credit pricing model. In fact, default probabilities are taken as inputs (while we are looking for a model to produce them as outputs) and it does not fully exploit market information about (past) equity and credit prices.

3.4.2 New information required: where to look for

Therefore, it seems that structural models need to incorporate the possibility of the firms defaulting because of liquidity shortages and high funding costs. The main problem is the difficulty for the credit risk modeler of getting data about the firm current and expected future cash flows, debt payments and funding costs. Where can they be found?

If the credit risk modeler is an investment banker employee working in the credit risk research department, he or she just has to go up (or down) a couple of floors to the fundamental equity analysis department to find people exclusively dedicated to predict firms' future cash flows, debt payments, equity values, ... So, the information is there! If the credit risk modeler is an academic, then he or she needs to have a friend in an investment bank ...

Imagine the fundamental equity analyst agrees to give its credit risk colleague the required information so he or she can incorporate it into the model. If the head of the fundamental equity analysis department hears about it, he or she will first be reluctant to give away information without any profit (that's why he or she is an investment banker). However, straight away he or she would (or should) think: if both departments are using similar information to price equity and credit products, why are they using different models? And the answer seems to be that there is no reason, which implies that they should be using a joint equity-credit model.

Both modelling approaches would benefit. The credit risk side would get information about the firms' asset values, cash flows, debt structure, funding costs, ... (and projections of them into the future). Note that such information is not directly composed of data taken directly from the firms' balance sheet (as it is the case in current structural models), but it is complemented with any other quantitative or qualitative information analysts have about the firms future prospects: managerial performance, the economic, political and competitive environment facing the company, as well as any current news or rumors relating to the company's operations.[6] On the other hand, from the credit risk desk the fundamental analysis team would get information about defaultable instruments of higher quality than the one they were already using.

Cash flows are the basis of equity modelling and, as argued above, they should play an important role in future structural models (as one of the motivations of defaults). As such, cash flows should drive equity-credit modelling. We have proposed two avenues of work in that direction in order to have a reliable equity-credit modelling framework: one theoretical (incorporating cash flows directly as a cause of default in existing structural models) and one empirical (merging the information sets used by credit modelers, mainly based on balance sheet information and market prices of defaultable and equity prices, and by equity modelers, built upon actual and projected cash flows).

To sum up, a reliable structural model for equity and debt pricing must (i) be theoretically improved to include the role of cash flows both in triggering default and in actual models for equity pricing, and (ii) incorporate all information that nowadays

[6] Some would argue that this information is already incorporated in market prices. Others would not.

credit research teams and fundamental equity analysts are using.

4 Can reduced form models play any role in equity-credit correlation modelling?

It does not seem easy.

The most obvious and straightforward approach would be to include equity prices as a risk factor in the default intensity of a firm and estimate its impact on credit risk together with other observable or unobservable factors. In fact, Chava and Jarrow (2004) show the higher performance of market variables, such as stock prices and volatility, over accounting variables when predicting defaults.

Chava and Jarrow (2004) estimate hazard rates using bankruptcy events and different observable variables. The authors use a logit model where the hazard rate (or default probability) of firm i at period t is given by

$$\frac{1}{1+e^{-\alpha_t-\beta' X_{it}}},$$

where X_{it} are time-varying variables (equity prices, accounting information, ...). Although not suitable for pricing purposes, their exercise sheds light on the relationship between equity prices and credit risk, as well as in the relative importance of equity prices vs. accounting variables in predicting defaults.

The authors use a database containing bankruptcy fillings between 1962-1999 and compare the predictive power of two models: one in which the X's contain both accounting and market (equity) variables and another using only market (equity) data. They obtain that the default predictive power of the simpler model, the one without accounting variables, is nearly identical to the one with accounting variables. Therefore, accounting variables seem not to add any extra information not included in equity prices and, moreover, their frequency is much lower (usually quarterly) than the one of equity data.

This evidence suggest the potential of reduced form models, using equity prices as risk factors in the default intensity.

However, including equity as a factor in the default intensity solves half of the problem: incorporating equity information when modelling credit risk; but not the other half: developing a model with information coming from equity and defaultable instruments to model equity prices and credit risk.

There have been attempts to pursue the joint modelling of equity and credit using reduced form models. However, the fact that the industry has not incorporated them yet as standard practices reveals the limitations of this approach.

Mamaysky (2002) and Dedov (2004) extend reduced form models considering models of joint bond and stock pricing, where both short rates and stock dividends are functions of a set of risk factors which capture the correlation structure. The authors provide closed form solutions for bond and equity prices. Although developed for default-free bonds, this framework is suitable for defaultable securities. However, the authors do not provide empirical implementations of the models.

The literature of convertible bonds pricing using reduced form models has included equity prices as one, if not the only one, of the variables determining default intensities.[7] Das, Sundaram and Sundaresan (2003) use a model incorporating equity, interest rate, liquidity and credit risk, where the credit risk component is based on a reduced form model where the default intensity depends on equity price and interest rates.

The main problem with the reviewed literature is that the models don't use the firms' fundamental asset and liability structure and, as a consequence, they still lack a coherent integration of equity and credit risk.

[7] See, among others, Takahashi et al. (2001), Davis and Lischka (2002), Andersen and Buffum (2003) and Ayache et al. (2003).

5 Concluding remarks

Some banks, such as JPMorgan, have recently combined equity and debt research, recognizing the synergies and information-sharing advantages of the move.[8] JPMorgan's head of credit research, G. Jenkins, puts it clear: "Seventy-five per cent of the work done by fundamental credit and equity analysts is the same."[9]

Our arguments in this report in favor of merging credit and equity research centres, in order to extend structural form models and use them as joint models for equity-credit pricing, completely agree with those put forward above.

We believe the prospects of structural models to become the theoretical and empirical framework for equity-credit modelling are high, something which can not be said about reduced form models.

The reluctance of most investment banks to change their actual structure (isolating credit and equity research) is an obstacle for the development of joint equity-credit models. Only when they feel the need to do so they will start thinking about possible solutions. Fortunately or unfortunately, the fact that we are already talking about it means it is being taken into account.

[8] See Credit (2005, p. 22 and 17).
[9] See Credit (2005, p. 22/23).

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